Hall Ticket Number:

Code No. : 13110 LAA

VASAVI COLLEGE OF ENGINEERING (*Autonomous*), HYDERABAD

B.E. (CBCS) III-Semester Main Examinations, December-2017

Linear Algebra and its Applications

Time: 3 hours Max. Marks: 70

Note: Answer ALL questions in Part-A and any FIVE from Part-B

Part-A (10 × 2 = 20 Marks)

1. Give any two examples of vector space.
2. Show that the set of matrices

   $S=\left\{\begin{bmatrix}-1 & 0\\ 2 & 1\end{bmatrix},\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}\right\}$ Does not span $M_{2\times2}$. Describe span (S).
3. Find $[V]_B$, given $B=\left\{\begin{bmatrix}3\\1\end{bmatrix},\begin{bmatrix}-2\\2\end{bmatrix}\right\}$; $V=\left\{\begin{bmatrix}8\\0\end{bmatrix}\right\}$
4. Let V and W be vector spaces, and let $T:V\rightarrow W$ be a linear transformation. Then show that $T(0) = 0$.
5. Define a linear operator $T: R^2\rightarrow R^2$ by $T\left(\begin{bmatrix}x\\y\end{bmatrix}\right)=\begin{bmatrix}2x-3y\\5x+2y\end{bmatrix}$. Show that T is one-to-one
6. Define linear transformation $T: P_4\rightarrow P_3$ by $T(p(x)) = p'(x)$. Find the null space and range of T.
7. Find the angle between the two vectors $u=\begin{bmatrix}2\\-2\\3\end{bmatrix}$ and $v=\begin{bmatrix}-1\\2\\2\end{bmatrix}$
8. Find proj_v **u.** Where $\mathbf{u}=\begin{bmatrix}3\\-2\end{bmatrix}$ and $\mathbf{v}=\begin{bmatrix}1\\-2\end{bmatrix}$
9. Find the scalar **c**, so that $\begin{bmatrix}-1\\c\\2\end{bmatrix}$ is orthogonal to $\begin{bmatrix}0\\2\\-1\end{bmatrix}$
10. Define Inner Product Vector Space.

Part-B (5 × 10 = 50 Marks)

(All sub-questions carry equal marks)

11. a) Write all 10 vector space axioms and show that $M_{2\times2}$ with the standard component wise operations is a vector space.

    b) Let W be the subspace of all symmetric matrices in the vector space $M_{2\times2}$.

    Let $B=\left\{\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix},\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix},\begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix}\right\}$

    show that B is a basis for W and find the coordinates of $v=\begin{bmatrix}2 & 3\\ 3 & 5\end{bmatrix}$ relative to B.

12\. a) Let $V = R^2$ with bases $B = \left\{\begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}1\\-1\end{bmatrix}\right\}$ and $B' = \left\{\begin{bmatrix}2\\-1\end{bmatrix}, \begin{bmatrix}-1\\1\end{bmatrix}\right\}$

Find the transition matrix from B to B'. Let $[\mathbf{v}]_B = \begin{bmatrix}3\\-2\end{bmatrix}$ and find $[\mathbf{v}]_{B'}$.

b) Let $\mathbf{v}_1 = \begin{bmatrix}1\\0\\-1\end{bmatrix}$ $\mathbf{v}_2 = \begin{bmatrix}0\\2\\21\end{bmatrix}$ $\mathbf{v}_3 = \begin{bmatrix}-3\\4\\7\end{bmatrix}$ and let $W = \textbf{Span}\ \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$.

i) Show that $\mathbf{v}_3$ is a linear combination of $\mathbf{v}_1$ and $\mathbf{v}_2$.

ii) Show that span $\{\mathbf{v}_1, \mathbf{v}_2\} = W$.

iii) Show that $\mathbf{v}_1$ and $\mathbf{v}_2$ are linearly independent.

13\. a) Define a Mapping T: $R^2 \to R^2$ by $T\left(\begin{bmatrix}x\\y\end{bmatrix}\right) = \begin{bmatrix}x+y\\x-y\end{bmatrix}$.

i) Find the image of the coordinate vectors e_1 and e_2 under the mapping T.

ii) Give a description of all vectors in R^2 that are mapped to the zero vector.

b) Define a linear transformation T: $P_2 \to P_3$ by $T(f(x)) = x^2 f''(x) - 2f'(x) + xf(x)$. Find the matrix representation of T relative to the standard bases for P_2 and P_3.

14\. a) Let V and W be finite dimensional vector spaces and $B = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\}$ a basis for V. *if* $T: V \to W$ be a linear transformation. Then R(T) = $\textbf{span}\{T(\mathbf{v}_1), T(\mathbf{v}_2), \ldots, T(\mathbf{v}_n)\}$,

b) Define T: $R^3 \to R^3$ by $T\left(\begin{bmatrix}x\\y\\z\end{bmatrix}\right) = \begin{bmatrix}x\\-y\\z\end{bmatrix}$.

i) Find the matrix of T relative to the standard basis for R^3.

ii) Use the result of part (a) to find $T\left(\begin{bmatrix}1\\1\\2\end{bmatrix}\right)$

15\. a) State and prove triangle inequality on vector spaces.

b) Let $u = \begin{bmatrix}1\\3\end{bmatrix}$ and $v = \begin{bmatrix}1\\1\end{bmatrix}$.

i) Find $\text{proj}_v\, \boldsymbol{u}$.

ii) Find $\mathbf{u} - \text{proj}_v\, \boldsymbol{u}$ and verify that $\text{proj}_v\, \boldsymbol{u}$ is orthogonal to $\mathbf{u} - \text{proj}_v\, \boldsymbol{u}$.

16\. a) If $S = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\}$ is an orthogonal set of nonzero vectors in an inner product space V, then prove that S is linearly independent.

b) Let V = P_2 with inner product defined by $\langle p, q\rangle = \int_{-1}^{1} p(x)q(x)dx$.

i) Show that the vectors in $S = \{1, x, \frac{1}{2}(3x^2 - 1)\}$ are mutually orthogonal.

ii) Find the length of each vector in S.

17. Answer any *two* of the following:

a) Let $S = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix} \right\}$ find the basis for span of S.

b) Let $T: R^3 \to R^3$ be a linear operator and B = {v_1, v_2, v_3} a basis for R^3. Suppose that

$$T(v_1) = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \qquad T(v_2) = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \qquad T(v_3) = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$$

i) Is $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ in R(T)?

ii) Find a basis for R(T).

c) Let $V = R^3$ and $u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$ and $v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$ be vectors in *V*. let *k* be a fixed positive real number, and define the function $\langle .,. \rangle : R^2 x R^2 \to R$ by $\langle u,v \rangle = u_1 v_1 + k u_2 v_2$. Show that *V* is an inner product space.

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